Wu Replies: The preceding Comment [1] on my Letter [2] and Haldane's Letter [3] cites three old papers by Anghel [4], which contained an unfounded claim that the thermodynamics derived in Ref. [2] for a generalized ideal gas, based on the generalized Pauli principle proposed in Ref. [3], is physically inconsistent. Reference [4], as well as [5], invoked the division of a species into subspecies in an attempt to claim that the mutual statistics parameters cannot be constant as proposed in [2,3]. In this reply I explain why the argument and the claim in Refs. [4,5] were at least seriously flawed and, therefore, failed to be a credible challenge to Refs. [2,3]. At the end of this Reply I explain why the Comment [1] still contains a serious misconception of subspecies division.

The generalized Pauli principle [3], or fractional exclusion statistics (FES) as later coined in Ref. [6], refers to the counting of many-body quantum states for quasiparticle excitations in certain strongly correlated condensed matter systems. A basic principle of quantum statistical mechanics (QSM) is the indistinguishability principle. Namely, the (quasi)particles in one given species (defined by a set of attributes or good quantum numbers) are identical or indistinguishable. Once the species are identified and counting of the quantum many-body states is done, the thermodynamics can be derived following the standard procedure of QSM [7], as done in Ref. [2], and contains no inconsistency. Division of a species into subspecies is unnecessary in QSM, and it has the danger of running directly into conflict with the indistinguishability principle of QSM. In Refs. [4,5], in an attempted challenge to Refs. [2,3], Anghel has invoked such a subdivision argument which, as I indicate below, has many loose ends and lacks a crucial consistency check for the thermodynamics to remain undisturbed.

The subdivision argument presented in Refs. [4,5] focuses on matching the change in the available single-particle states before and after the subdivision. Several important issues have not been addressed at all about the subdivision (say, of species 1 into two species $1a$ and $1b$). For the numbers, G_{1a} and G_{1b}, of single-particle states of species $1a$ and $1b$ in the absence of all other particles, respectively, even with a suggested constraint $G_1 = G_{1a} + G_{1b}$, there are arbitrariness or ambiguities in assigning the value of G_{1a}: Should it be fixed or not? If G_{1a} is fixed, what is the principle to select that value? If not, how should one deal with all possible cases? The more fundamental issue is how to guarantee that the total statistical weight, which is a product over all subspecies, is unchanged by subdividing some species. This is needed for consistency so as not to disturb the thermodynamics of the system. Both Refs. [4,5] failed to address these key issues. It is no wonder that a claim was made in [4,5] that a thermodynamics different from that of Ref. [2] could be "derived" with the subdivision argument, without having checked the total many-body state counting.

In the preceding Comment [1], the rules for the FES parameters in Refs. [2,3] are said to be "arbitrary." On one hand, Anghel has verified in Ref. [8] that his result, Eq. (2) in the Comment [1], for one-dimensional exactly solvable statistical models actually agrees with mine in Ref. [9] using the method of [2]. On the other hand, the FES parameters in [2,3] are obviously not arbitrary, since the FES parameters in this case have been derived in Ref. [9] from the Bethe ansatz equations, which were used as the starting point in Ref. [8] as well [see Eq. (19) in [8]]. Moreover, as correctly recognized in Ref. [8], the cases of two-dimensional fractional quantum Hall systems, in which the numerical tests provided in Ref. [10] support constant (i.e., G_i-independent) mutual FES parameters, are exceptions to the rules conjectured in Refs. [4,5].

The Comment [1], and all the papers in [4,5,8], share a common misconception of subspecies. In standard state counting, there is a separate statistical weight factor for each species. If one liked to split a species into subspecies, the number of weight factors should increase, one for each subspecies. This was not done in [1,4,5,8]. So what was really done there had nothing to do with species splitting or coarse graining but rewriting the FES parameters as a sum of two or more terms. This is arbitrary and unnecessary, trivially not changing state counting and thermodynamics, and cannot always be done [8,10].

In conclusion, the "species subdivision" presented in the Comment [1] and Refs. [4,5,8] does not represent an adequate and valid argument to challenge my Letter [2].

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